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In the framework of a (2+1)-dimensional P-even massive Gross-Neveu model, an external magnetic field is shown to induce a parity breaking first order phase transition. Possibility of applying the results obtained to description of magnetic phase transitions in high-temperature superconductors is discussed.

I. INTRODUCTION

The dynamical symmetry breaking phenomenon induced by external magnetic or chromomagnetic fields is called magnetic catalysis effect. This property of a uniform external magnetic field H was for the first time observed in the study of (2+1)-dimensional (3-d) chiral invariant field theories with four-fermion interaction (the so called Gross – Neveu (GN) type theories [1]). In this case, an arbitrary weak external magnetic field leads to dynamical chiral symmetry breaking (DχSB) even for an arbitrary small coupling constant [2,3]. This phenomenon was then explained basing upon mechanism of effective reduction of space-time dimensions in the external magnetic field and, corresponding strengthening of the role of infrared divergences in the vacuum reorganization [4]. Later, it has been shown that spontaneous chiral symmetry breaking can be induced by an external chromomagnetic field as well [5–7]. Moreover, basing upon the study of a number of field theories, it has been argued that DχSB magnetic catalysis effect may have a universal, i.e. model independent, character [8] (for 3-d quantum field theories this fact was proved in [4]). In a recent paper [9] in the framework of a $P-$even 3-d GN model, it has been demonstrated that the external magnetic field serves as a catalyst for spontaneous parity breaking as well. Magnetic catalysis has already found its applications in cosmology and astrophysical investigations [10], and also in constructing the theory of high-temperature superconductivity [9,11–13]. It can be positively stated that this phenomenon will find its application in the elementary-particle physics, condensed-matter physics, physics of neutron stars etc., i.e., in those branches of science, where the dynamical symmetry breaking plays a crucial role, and an external magnetic field is present. [1] For the last ten years, 3-d field theories including GN type models [15] have attracted a considerable interest. In many respects, this can be explained by the fact that high-temperature superconductivity (HTSC) is a planar phenomenon, i.e., conduction electrons in HTSC materials are concentrated in planes formed by atoms of Cu and O [16]. In recent experimental studies of high temperature superconductor $Bi_2Sr_2CaCu_2O_8$ [17], it was discovered that thermal conductivity as a function of an external magnetic field H experiences a jump at a certain value of $H = H_c \sim T^2$ (the sample temperature $T < T_c$, where T_c is the temperature of transition to the superconducting state). The authors of [17] assumed that at $H = H_c$ a phase transition induced by a magnetic field takes place. Phenomenological description of this phase transition (based upon the free energy functional of the system) as a first order parity breaking transition was soon proposed

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[1] Possible applications of the magnetic catalysis effect were also discussed in recent publications [14].

[18]. Moreover, a theoretical (microscopic) explanation of this phenomenon in the framework of two 3-d models of the GN type based upon the magnetic catalysis effect of the dynamical symmetry breaking were presented [9,11] [2]. In one of them, a magnetic field induces dynamical breaking of chiral symmetry [11], in the other, a parity breaking phenomenon takes place [9]. A general feature of phase transitions for these two models is their continuity. In other words, both chiral symmetry in [11] and parity in [9] are violated at the point $H = H_c$ by means of the second order phase transition, and this does not agree with the phenomenological approach of [18].

In the present paper, we show that there exists a (2+1)-d GN model, where the magnetic catalysis demonstrates the same qualitative properties, as in phenomenological description of a phase transition in HTSC systems induced by an external magnetic field [18].

II. STATEMENT OF THE PROBLEM

Consider the influence, an external magnetic field exerts on the phase structure of the (2+1)-d GN model, described by the Lagrangian

$$L = \sum_{a=1}^{N}(\bar{\psi}_a i\hat{\partial}\psi_a + m\bar{\psi}_a\psi_a) + \frac{G}{2N}(\sum_{a=1}^{N}\bar{\psi}_a\tau\psi_a)^2, \tag{1}$$

where $\hat{\partial} \equiv \gamma^\mu\partial_\mu$, the fields ψ_a are transformed according to the fundamental representation of the $U(N)$ group introduced in order to employ the nonperturbative $1/N$-expansion method. Moreover, for every value of $a = 1, 2, ..., N$, Fermi fields ψ_a are four-component Dirac spinors, reducible with respect to the Lorentz group (corresponding indices are omitted). The matrix τ in the spinor space has the form: $\tau = \mathrm{diag}(1, 1, -1, -1)$. The action with the Lagrangian (1) is invariant under the discrete parity (P) transformation $\psi_a(t, x, y) \to i\gamma^1\gamma^5\psi_a(t, -x, y)$ (the algebra of γ^μ matrices for the reducible 4-dimensional spinor representation of the Lorentz group for 3-d space-time is presented in [20].) It is this model at $m = 0$ that was used in [9] for the description of phase transitions induced by an external magnetic field in HTSC systems. We demonstrate that, in contradistinction to the case of $m = 0$, in this model at $m \neq 0$ at the critical point $H = H_c$, the external magnetic field induces dynamical breaking of parity that takes place discretely, i.e., in the framework of the first order phase transition. This property of the magnetic catalysis in the model (1) at $m \neq 0$ has no analogy in any other field model, and this is demonstarated for the first time in the present paper. With the results obtained, one may hope that the massive GN model (1) can be used for theoretical explanation of the magnetic phase transition in the experiment [17], in agreement with the phenomenological approach [18].

III. PHASE STRUCTURE OF THE MODEL AT $H = 0$

Before considering the influence of the nonzero external magnetic field H on the vacuum of the model (1), its phase structure will be studied at $H = 0$. To this end, we introduce the auxiliary Lagrangian

$$\tilde{L} = \bar{\psi}i\hat{\partial}\psi + m\bar{\psi}\psi + \sigma\bar{\psi}\tau\psi + \frac{N\sigma^2}{2G}, \tag{2}$$

[2] The relevance of other 3-d relativistic models to this phase transition was recently discussed in [19].

where $\sigma(x)$ is an auxiliary real boson field, and summation over indices a of the auxiliary group $U(N)$ is implicitly performed here and in what follows. The field theories (1) and (2) are equivalent, since the field σ can be excluded from (2) by means of the equations of motion and the Lagrangian (1) is obtained. It is easily shown that the auxiliary field undergoes discrete parity transformation P in the following way: $\sigma(t,x,y)\to-\sigma(t,-x,y)$, i.e. σ is a pseudoscalar field. Starting from the Lagrangian (2), one can find the effective potential of the theory, which has the following form in the one-loop approximation (i.e. in the leading order of the $1/N$ expansion):

$$V_0(\sigma)=\frac{N\sigma^2}{2G}-N\sum_{k=1}^{2}\int\frac{d^3p}{(2\pi)^3}\ln(p^2+M_k^2), \tag{3}$$

where integration is performed over the Euclidean momentum, and $M_{1,2}=\mid m\pm\sigma\mid$. Integration in (3) over the domain $0\le p^2\le\Lambda^2$ yields:

$$\frac{1}{N}V_0(\sigma)=\frac{\sigma^2}{2g}+\frac{|m-\sigma|^3}{6\pi}+\frac{|m+\sigma|^3}{6\pi}, \tag{4}$$

where: $\frac{1}{g}=\frac{1}{G}-\frac{2\Lambda}{\pi^2}$. It is well known that the phase structure of any theory is to a great extent determined by the symmetry of the global minimum of its effective potential. It is easily shown that the function (4) has a global minimum point $\sigma=0$ for $\pi/g>-2m$, symmetric with respect to parity transformation. If $\pi/g<-2m$, potential (4) obtains in the domain $0\le\sigma<\infty$ a nontrivial absolute minimum at the point

$$\sigma_0=-\frac{\pi}{2g}+\sqrt{\frac{\pi^2}{4g^2}-m^2}, \tag{5}$$

and parity of the model is spontaneously broken. When we use a bare coupling constant G insted of g, the following picture is obtained. At $G<G_c=\pi^2/(2\Lambda-2m\pi)$ the vacuum of the model is $P-$even, at $G>G_c$ a phase of the model with spontaneously broken parity is realized. At the point $G=G_c$ the global minimum of the potential jumps from the origin to the nontrivial point (5), and the first order phase transition takes place. (We note for comparison that in the case of a massless theory, i.e., for $m=0$, a second order phase transition continuous in the coupling constant takes place at the point $G=G_c$.)

IV. MAGNETIC CATALYSIS OF DYNAMICAL PARITY BREAKING

We now consider the influence of an external magnetic field H (at zero temperature) on the symmetric phase of the model (1), i.e., in the case $\pi/g>-2m$ (the bare coupling constant is sufficiently small in that case: $G<G_c$). The corresponding effective potential, which is a special case of the effective potential for a more general GN model at $H\neq0$ [20], has the form:

$$V_H(\sigma)=\frac{N\sigma^2}{2g}+N\sum_{k=1}^{2}\left[\frac{eHM_k}{4\pi}-\frac{(2eH)^{3/2}}{4\pi}\zeta\left(-\frac{1}{2},\frac{M_k^2}{2eH}\right)\right], \tag{6}$$

where $\zeta(s,x)$ is the generalized Riemann zeta-function [21]. Let us demonstrate that an external magnetic field induces spontaneous parity breaking for this model as well. Moreover, the magnetic catalysis phenomenon and its properties being qualitatively different for cases $m=0$ and $m\neq0$.

The case with $m=0$. The corresponding effective potential of the model $V_H^{m=0}(\sigma)$ is obtained from the formula (6) at $m=0$:

$$V_H^{m=0}(\sigma) = \frac{N\sigma^2}{2g} + \frac{NeH|\sigma|}{2\pi} - \frac{N(2eH)^{3/2}}{2\pi}\zeta\left(-\frac{1}{2}, \frac{\sigma^2}{2eH}\right). \tag{7}$$

It is clear that $V_H^{m=0}(\sigma)$ is a function symmetric under the transformation $\sigma \to -\sigma$. Hence, in order to find a global minimum point, its investigation only in the set $\sigma \in [0, \infty)$ will suffice. In this case the stationary equation

$$\frac{\partial V_H^{m=0}(\sigma)}{\partial\sigma} \equiv 2N\sigma F(\sigma) = 2N\sigma\left[\frac{1}{2g} + \frac{eH}{4\pi\sigma} - \frac{\sqrt{2eH}}{4\pi}\zeta\left(\frac{1}{2}, \frac{\sigma^2}{2eH}\right)\right] = 0 \tag{8}$$

is obtained from (7) with the help of the formula $d\zeta(s, x)/dx = -s\zeta(s+1, x)$. The potential (7) was studied in [2,4,6]. Nevertheless, we will dwell upon some details that we will need in what follows. First of all, we point out that it follows from (8) that

$$\left.\frac{\partial V_H^{m=0}(\sigma)}{\partial\sigma}\right|_{\sigma\to 0_+} = \lim_{\sigma\to 0_+}(2N\sigma F(\sigma)) = -\frac{NeH}{2\pi}, \tag{9}$$

i.e., the point $\sigma = 0$ is not a solution of the stationary equation (8). Moreover, since $V_H^{m=0}(\sigma) = V_H^{m=0}(-\sigma)$, there appear two more consequences of the formula (9): 1) At the pont $\sigma = 0$, a local maximum of the potential is situated. 2) The first derivative of the function $V_H^{m=0}(\sigma)$ does not exist at the point $\sigma = 0$. (At the same time, at $H = 0$, the effective potential is a differentiable function on the whole $\sigma-$axis.) Consequence 1), as well as the fact that $\lim_{|\sigma|\to\infty} V_H^{m=0}(\sigma) = +\infty$, indicate the presence of a nontrivial global minimum of potential $V_H^{m=0}(\sigma)$ at the point $\sigma_0(H) \neq 0$. This means that parity of the massless model under consideration in the presence of an external magnetic field (arbitrary small) is inevitably spontaneously broken even for arbitrary small coupling constant G (in this case $g > 0$). Hence, a dynamical breaking of parity, induced by an external magnetic field takes place (magnetic catalysis phenomenon).

It is known that $F(\sigma)$ is a monotonically increasing function in the set $\sigma \in [0, \infty)$, so that $F(0) = -\infty$ (the consequence of equality (9)) and $\lim_{\sigma\to\infty} F(\sigma) = \infty$ [2,6]. Hence, there exists a unique point $\sigma_0(H) \neq 0$, where the function $F(\sigma)$ vanishes, and this is the place, where the global minimum of potential (7) is situated. It follows from the equation $F(\sigma) = 0$, implicitly defining the function $\sigma_0(H)$, that $\sigma_0(H) \approx egH/(2\pi)$ at $H \to 0$ [2,6]. *Thus, when a magnetic field is included in the model, a parity breaking second order phase transition continuous in H takes place, due to the fact that the order parameter $\sigma_0(H)$ is a continuous function of H at the point of phase transition (i.e., at $H = 0$).*

It was proved earlier [2,6] that at large values of H the function $\sigma_0(H)$ behaves as follows: $\sigma_0(H) \approx k\sqrt{eH}$, where k is a solution of the equation $1 = \sqrt{2}\zeta(1/2, k^2/2)$ and $k \approx 0.45$. As $\sigma_0(H)$ is the point of global minimum of potential (7), the following inequality is valid for all values of H: $V_H^{m=0}(0) > V_H^{m=0}(\sigma_0(H))$. In particular, using this relation for $H \to \infty$ renders

$$-\frac{N(2eH)^{3/2}}{2\pi}\zeta\left(-\frac{1}{2}, 0\right) > \frac{Nk(eH)^{3/2}}{2\pi} - \frac{N(2eH)^{3/2}}{2\pi}\zeta\left(-\frac{1}{2}, \frac{k^2}{2}\right). \tag{10}$$

The case with $m \neq 0$. We will here discuss some special features of spontaneous symmetry breaking in the model (1) at $H, m \neq 0$. As in the massless case, at $m \neq 0$ potential $V_H(\sigma)$ is an even function of σ, and hence, suffices to study it on the semiaxis $\sigma \in [0, \infty)$. Here, the stationary equation for the effective potential has the form:

$$0 = \frac{1}{N}\frac{\partial V_H}{\partial\sigma} = \begin{cases} (\sigma+m)F(\sigma+m) - (m-\sigma)F(m-\sigma), & \sigma < m; \\ (\sigma+m)F(\sigma+m) + (\sigma-m)F(\sigma-m), & \sigma > m, \end{cases} \tag{11}$$

where function $F(x)$ is presented in (8). With regard for (9), it is easily obtained from the above equation

$$\frac{\partial V_H}{\partial \sigma}\bigg|_{\sigma\to m_+} = -\frac{NeH}{2\pi} + \frac{\partial V_H}{\partial \sigma}\bigg|_{\sigma\to m_-}. \tag{12}$$

This means that at $H \neq 0$ potential (6) is not differentiable at points $\sigma = \pm m$. By virtue of this, the possible global minimum point of the function $V_H(\sigma)$ in the set $\sigma \in [0, \infty)$ is either at the point $\sigma = m$, or at one of the solutions of the stationary equation (11). However, using (12), one can make the important conclusion: if, at a certain vicinity to the left of the point $\sigma = m$, the derivative of the effective potential is negative, its derivative in a certain vicinity to the right of it is also negative. Hence, $\sigma = m$ is unable to be not only the global minimum point, but even a local minimum point. Therefore, special attention should be paid to solutions of equation (11). An evident solution of this equation for all values of H is the pont $\sigma = 0$. By means of both analytical and numerical methods, one can demonstrate that for all $H \neq 0$ there exists at this point at least a local minimum of the function $V_H(\sigma)$. Moreover, for sufficiently small values of H the global minimum of the potential is situated just at the point $\sigma = 0$, and parity remains unbroken (this is the consequence of the fact that with such H the derivative of the potential is positive for all $\sigma \in [0, \infty)$).

We will demonstrate that at large H the equation (11) has one more solution that is absent for small H. Indeed, in the domain $\sigma >> m$, this equation coincides in form with the stationary equation (8) for the massless case. The solution $\sigma_0(H)$ of the latter becomes large only in the limit $eH \to \infty$. Hence, in the region of large values of the magnetic field, equation (11) has, besides $\sigma = 0$, one more solution $\tilde{\sigma}_0(H)$, such that $\tilde{\sigma}_0(H) \approx k\sqrt{eH}$ at $eH \to \infty$ (the value of coefficient k is the same as in (10)). Besides $\sigma = 0$ and $\tilde{\sigma}_0(H)$, we were able to find no more solutions of the stationary equation (11).

As it was already pointed out above, at sufficiently small H, the global minimum of the potential is situated at the point $\sigma = 0$. We will demonstrate that with growing H it goes over to the point $\tilde{\sigma}_0(H)$. To this end, we will find the values of the potential in stationary points for $eH/m^2 \to \infty$:

$$V_H(0) = -\frac{(2eH)^{3/2}}{2\pi}\zeta\left(-\frac{1}{2}, 0\right) + O(eH/m^2), \tag{13}$$

$$V_H(\tilde{\sigma}_0(H)) = \frac{k(2eH)^{3/2}}{2\pi} - \frac{(2eH)^{3/2}}{2\pi}\zeta\left(-\frac{1}{2}, \frac{k^2}{2}\right) + O(eH/m^2), \tag{14}$$

where we considered that $\tilde{\sigma}_0(H) \approx k\sqrt{eH}$ for $eH \to \infty$. Comparing expressions (13) and (14) with the help of (10), we come to the conclusion that $V_H(\tilde{\sigma}_0(H)) < V_H(0)$ at $eH \to \infty$. Since for all values of H, $\sigma = 0$ is at least a local minimum, the passing of a global minimum of the potential from the point $\sigma = 0$ to the nontrivial point $\tilde{\sigma}_0(H)$ takes place in a jump at a certain critical value of an external magnetic field $H_c(g) \neq 0$. *Thus, at the point $H_c(g)$ a parity breaking first order phase transition takes place.* The critical magnetic field $H_c(g)$ as a function of the coupling g at $T = 0$ and $m \neq 0$ is depicted in Fig. 1. In Fig. 2, the functions $V_H(\sigma)$ are plotted at various values of H and $gm = 10$.

V. MAGNETIC CATALYSIS AT NONZERO TEMPERATURE

Employing the technique developed in [4], one can obtain the following expression for the effective potential of the model (1) at $H, T \neq 0$:

$$\frac{1}{N}V_{HT}(\sigma)=\frac{1}{N}V_H(\sigma)-\frac{eH}{2\pi\beta}\sum_{i=1}^{2}\{\ln(1+\exp(-M_i\beta))+2\sum_{k=1}^{\infty}\ln(1+\exp(-\beta\sqrt{M_i^2+2keH}))\}, \quad (15)$$

where $\beta=1/T$, and $V_H(\sigma)$ is the potential (6). Numerical analysis of the potential (15) demonstrates that also in this case parity is unbroken at sufficiently small values of H. However, there exists a critical magnetic field, when a parity breaking first order phase transition appears. At fixed value of g, we denote the critical field value as $H_c(T)$. Some of its values for $gm=5$ are presented in the Table 1. From this Table it is seen that $H_c(T)\sim T^2$ at $T\to\infty$, i.e., $H_c(T)$ behaves as in models [9,11].

VI. CONCLUSIONS

In the present paper, it was proved, in the framework of quantum field theory, that an external magnetic field can induce a first order phase transition. As an illustration, we proposed a $P-$even massive 3-d GN model (1) [3].

At $m=0$, magnetic catalysis of the dynamical parity breaking takes place in this model [9]. Moreover, at the point $H_c\sim T^2$ $(T\to\infty)$, there appears *a second order phase transition*, and theoretical values H_c provide satisfactory description of the experimental data [17].

We have shown that, in the model (1) at $m\neq 0$, dynamical parity breaking is also induced by an external magnetic field. The curve H_c at large T behaves in the same way as at $m=0$. However, in contradistinction to the latter case, in the massive GN model (1) at the point H_c, there appears a parity breaking *first order phase transition*, and this fact is in agreement with the phenomenological description [18] of magnetic phase transitions in HTSC systems in the experiment [17].

VII. ACKNOWLEDGMENTS

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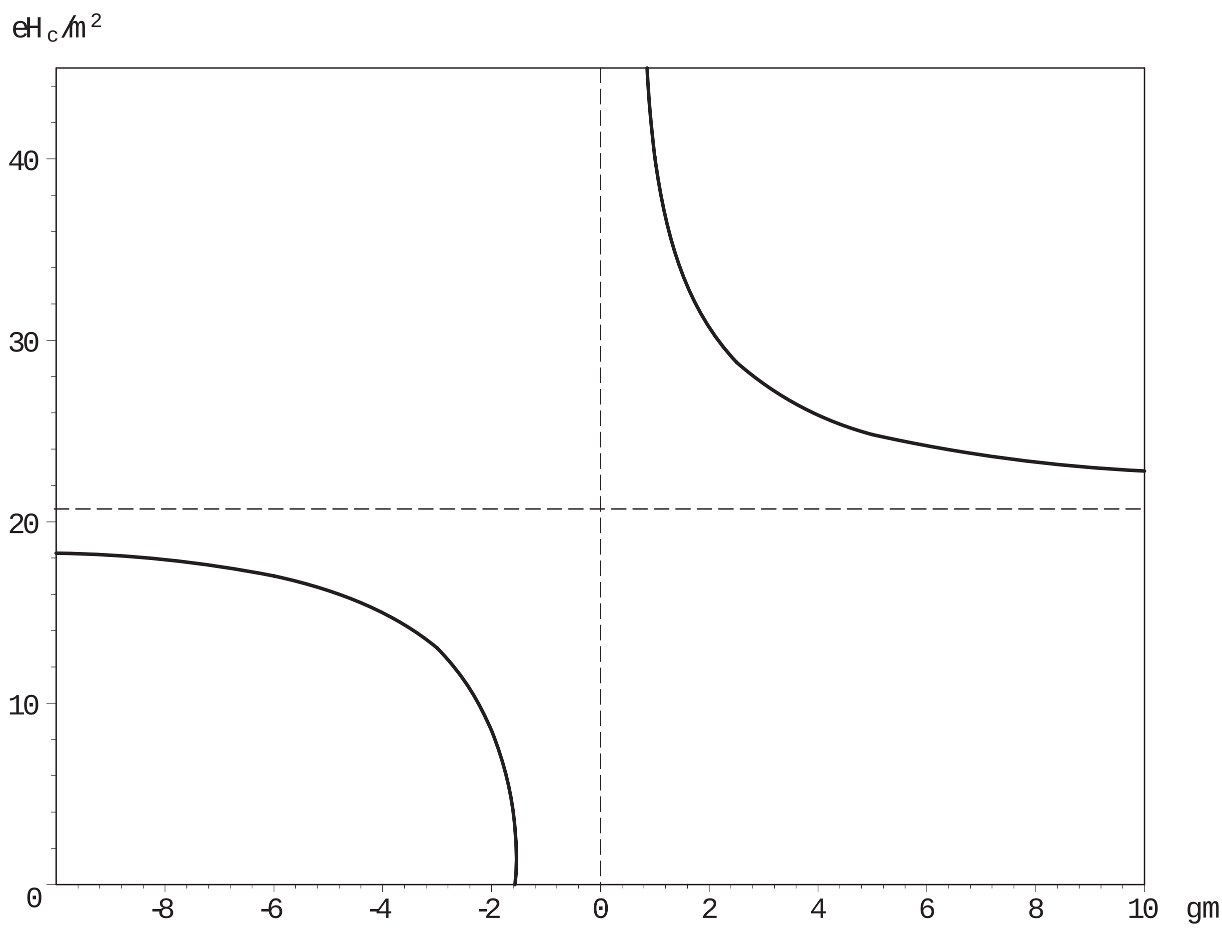

FIG. 1. Critical magnetic field as a function of coupling constant g

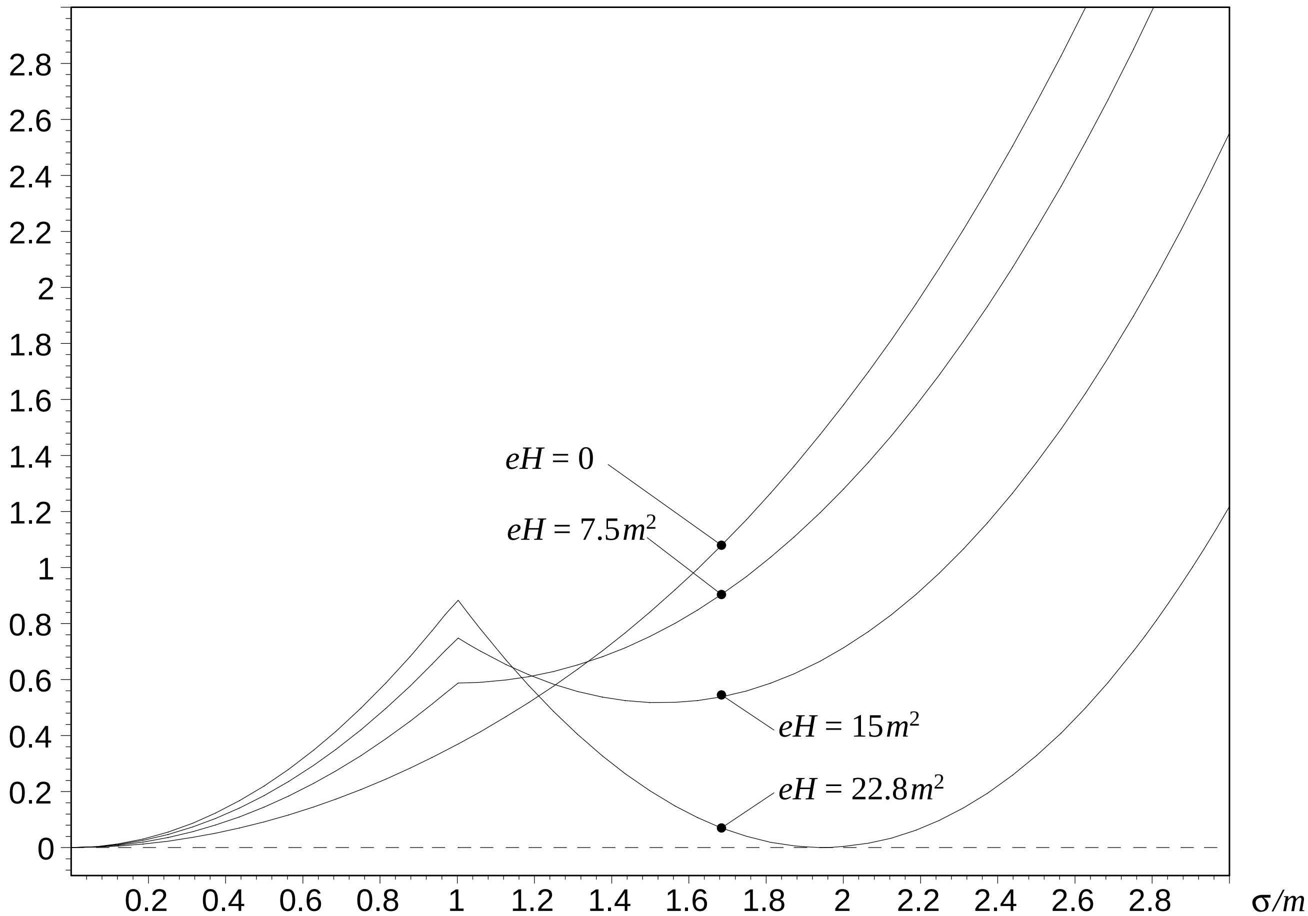

FIG. 2. Effective potential at various values of magnetic field intensity and $gm = 10$ ($\sigma/m = x$).

TABLE I. Some values of $H_c(T)$ at $gm = 5$.

T/m	0.1	0.25	0.5	1	2.5	5	10	25	50	100	200
$eH_c(T)/m^2$	24.9	25.2	27	36	131	465	1782	10900	43200	173000	700000